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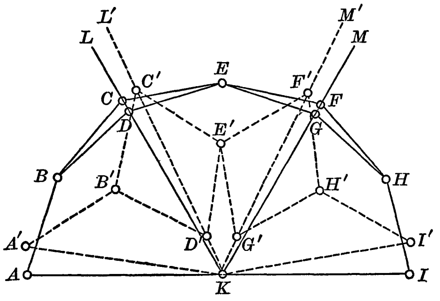
The last result was incorrectly stated by the Proposer.

Also solved by Herbert N. Carleton, J. A. Caparo, F. L. Griffin, Frank R. Morris, G. W. Hartwell, C. E. Horne, H. C. Feemster, Horace Olson, J. V. Balch, H. L. Agard, and the Proposer.

GEOMETRY.

427. Proposed by F. CAJORI, Colorado College.

In S. Gross's linkage for trisection of angles, shown in the figure (KL' and KM' being the trisectors of $A'KI'$), C is fixed on KL, also F on KM; at starting, C and D coincide, also F and G; D slides on KL, G slides simultaneously on KM; if E moves along a perpendicular to AI erected at K, find the loci of B and D.



Solution by J. W. Clawson, Collegeville, Pa.

Denote length of KC by b, $EC = CB = BA$ by a. Now $KA = b$, and, projecting the broken line $KCBA$ on KA,

$$b \cos \pi/3 + a \cos \pi/4 + a \cos 5\pi/12 = b.$$

Hence

$$b = 2a \cos \pi/12.$$

Denote length of KD' at any time by r, angle EKD' by θ, angle $E'C'K = E'D'C'$ by ψ. Then $r + 2a \cos \psi = b$. But, by the law of sines,

$$\frac{\sin (\theta + \psi)}{\sin \theta} = \frac{b}{a}.$$

Solving for $\cos \psi$,

$$\cos \psi = \frac{b}{a} \sin^2 \theta \pm \frac{\cos \theta}{a} \sqrt{a^2 - b^2 \sin^2 \theta}.$$

Hence,

$$r - b + 2b \sin^2 \theta = \mp 2 \cos \theta \sqrt{a^2 - b^2 \sin^2 \theta},$$

i. e.,

$$(r - b \cos 2\theta)^2 = 4 \cos^2 \theta (a^2 - b^2 \sin^2 \theta).$$

This is the polar (r, θ) equation of the locus of D' with respect to KE as initial line and K as pole.

Again, by the law of cosines, $KB'^2 = b^2 + a^2 - 2ab \cos \psi$. Denote length of KB' by ρ and $\angle EKB'$ by ϕ. The equation just written becomes

$$(\rho^2 - a^2 - b^2 \cos \phi)^2 = 2b^2(1 + \cos \phi) \left\{ a^2 - b^2 \frac{1 - \cos \phi}{2} \right\}.$$

This is the polar (ρ, ϕ) equation of the locus of B' where KE is initial line and K is pole.

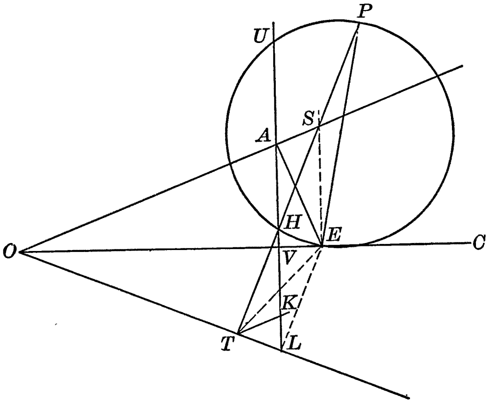
463. Proposed by NATHAN ALTSHILLER, University of Washington, Seattle.

Through a given point, to draw a line that cuts off on the sides of a given angle two segments the sum of which has a given value.

Solution by J. A. Caparo, University of Notre Dame.

Let $\angle O$ be the given angle, P the given point, $OA = k$ half the required length.

Construction. Let OC bisect the given angle. Draw $AE \perp$ to OA and $AV \perp$ to OC, thus making $OL = OA$. On PE as a diameter describe a circle intersecting AL at U and H. Then PH and PU are the required lines.



Proof. Draw ES, ET and draw EL which will be $\perp$ to OL. The quadrilaterals $EHAS$ and $EHTL$ are inscriptible in circles since SAE and EHS; EHT and ELT are right angles.
Hence,

$$\angle SEA = \angle AHS = \angle THL = \angle TEL \quad \text{and} \quad \triangle ELT = \triangle EAS.$$

Then $ES = ET$, $HS = HT$ and $AS = TK = TL$, where TK is parallel to OA.
But, $OA + OL = 2k$ by construction. Hence,

$$OA + AS + OL - AS = 2k \quad \text{or} \quad OA + AS + OL - TL = 2k,$$

and finally,

$$OS + OT = 2k.$$

The line PU produced cuts from the given angle segments whose *difference* is $2k$. The proof is the same as the above.

An excellent solution was given by Professor F. L. Griffin, of Reed College, the proof of which involves the calculus.

Using the same construction as in Professor Caparo's solution, he notes the fact that, after the point E is located, the rest is merely the well known construction of a tangent to a parabola from a given point. This parabola, which has OC as its axis, E as its focus, and AL as the tangent at its vertex, is the envelope of the whole family of lines cutting off on the sides of LO two segments whose sum is $2k$. Thus, the required line is merely that tangent to this parabola which passes through the given point P.

This point of view brings out the discussion that there is no solution, one solution, or two solutions, according as the given point P lies within, on, or without the parabola. If P lies outside the given angle, as in the figure above, then one of the lines, as PU, will cut one side of the angle produced through the vertex, but the sum will still be $2k$ if we call this segment negative. Professor Griffin also gives a solution involving analytic geometry but not the calculus. Others may be interested to work out both of these solutions. Editors.

Also solved by A. H. Holmes, Paul Capron, J. W. Clawson, and N. P. Pandya.

468. Proposed by ELMER SCHUYLER, Brooklyn, New York.

Given two circles and a straight line, to draw a circle tangent to the line and coaxial with the two given circles.